Errata and Hints for "Quantum Walks and Search Algorithms" (2nd edition) by Renato Portugal, Springer, Cham/Switzerland, 2018

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Chapter 3

1. Exercise 3.12 on page 38. Some hints on how to prove the identity (for $n \geq 0$)

$$\mathrm{e}^{-2\,\mathrm{i}\,\gamma\,t} J_{|n|}\left(2\,\gamma\,t\right) = \mathrm{e}^{\frac{\pi \mathrm{i}}{2}|n|} \sum_{k=|n|}^{\infty} \frac{\left(-\mathrm{i}\gamma t\right)^{k}}{k!} \binom{2\,k}{k-n}.$$

The first step is to change the dummy index $k \to \ell + n$ so that the sum starts from $\ell = 0$:

$$\mathrm{e}^{-2\,\mathrm{i}\,\gamma\,t} J_{|n|}\left(2\,\gamma\,t\right) = \mathrm{e}^{\frac{\pi \mathrm{i}}{2}|n|} \sum_{\ell=0}^{\infty} \frac{\left(-\mathrm{i}\gamma t\right)^{\ell}}{\ell!} \left(-\mathrm{i}\gamma t\right)^{n} \binom{2\,(\ell+n)}{\ell} \frac{\ell!}{(\ell+n)!}.$$

The second step is to change t to $t' = -\mathrm{i}\gamma t$:

$$\frac{\mathrm{e}^{2t'} J_{|n|}\left(2\,\mathrm{i}\,t'\right)}{(t')^{n} \mathrm{e}^{\frac{\pi \mathrm{i}}{2}|n|}} = \sum_{\ell=0}^{\infty} \binom{2\,(\ell+n)}{\ell} \frac{\ell!}{(\ell+n)!} \frac{(t')^{\ell}}{\ell!}.$$

To prove the identity (first formula), we have to show that the right-hand side of the above equation is the Taylor expansion of the left-hand side, that is, we have to show that

$$\left.\frac{\mathrm{d}^{\ell}}{\mathrm{d}t'^{\,\ell}} \left(\frac{\mathrm{e}^{2t'} J_{|n|}\left(2\,\mathrm{i}\,t'\right)}{(t')^{n} \mathrm{e}^{\frac{\pi \mathrm{i}}{2}|n|}}\right)\right|_{t'=0} = \binom{2\,(\ell+n)}{\ell} \frac{\ell!}{(\ell+n)!}.$$

The next step is to use the Taylor expansion of $J_{|n|}\left(2\,\mathrm{i}\,t'\right)$ on the left-hand side and to proceed with the calculations.

Chapter 4

1. Exercise 4.19. $m = \frac{N}{2} \longrightarrow m = \frac{N}{4}$ and $m = \frac{N}{4} \longrightarrow m = \frac{N}{8}$

Chapter 8

1. In the last line of page 170, the expression $\mathrm{e}^{\mathrm{i}kx}\left|\tilde{\psi}_{\ell}^{k'}\right\rangle = \left|\tilde{\psi}_{\ell}^{(k'-k)}\right\rangle$ must be replaced by $\mathrm{e}^{\mathrm{i}kX}\left|\tilde{\psi}_{\ell}^{k'}\right\rangle = \left|\tilde{\psi}_{\ell}^{(k'-k)}\right\rangle$. Note that $\mathrm{e}^{\mathrm{i}kX}$ is an operator and its action on the computational basis is $\mathrm{e}^{\mathrm{i}kX}|x\rangle = \mathrm{e}^{\mathrm{i}kx}|x\rangle$.

Chapter 9

1. Exercise 9.11 page 189. The expression

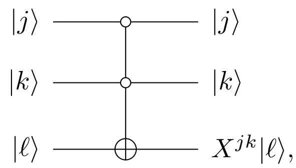
$$S_N \;=\; \sum_{\substack{k,\ell=0 \\ (k,\ell)\neq(0,0)}}^{\sqrt{N}-1} \frac{1}{1-\frac{1}{2}\left(\cos\frac{2\pi k}{\sqrt{N}} - \cos\frac{2\pi \ell}{\sqrt{N}}\right)}$$

must be replaced by

$$S_N = \sum_{\substack{k,\ell=0 \\ (k,\ell)\neq(0,0)}}^{\sqrt{N}-1} \frac{1}{1-\frac{1}{2}\left(\cos\frac{2\pi k}{\sqrt{N}}+\cos\frac{2\pi \ell}{\sqrt{N}}\right)}$$

Appendix A

1. Page 267. The circuit



must be replaced by