Decoherence in Two-Dimensional Quantum Walks

A.C. Oliveira, R. Portugal*
Laboratório Nacional de Computação Científica, LNCC, C.P. 95113, Petrópolis, RJ, 25651-075, Brazil

R. Donangelo
Universidade Federal do Rio de Janeiro, UFRJ, C.P. 68528, Rio de Janeiro, RJ, 21941-972, Brazil
(Dated: June 9, 2006)

We analyze the decoherence in quantum walks in two-dimensional lattices generated by broken-link-type noise. In this type of decoherence, the links of the lattice are randomly broken with some given constant probability. We obtain the evolution equation for a quantum walker moving on 2-D lattices subject to this noise, and we point out how to generalize for lattices in more dimensions. In the non-symmetric case, when the probability to break links in one direction is different from the probability in the perpendicular direction, we have obtained a non-trivial result. If one fixes the link-breaking probability in one direction, and gradually increases the probability in the other direction from 0 to 1, the decoherence initially increases until it reaches a maximum value, and then it decreases. This means that, in some cases, one can increase the noise level and still obtain more coherence. Physically, this can be explained as a transition from a decoherent 2-D walk to a coherent 1-D walk.

PACS numbers: 03.67.Lx, 03.65.Yz, 05.40.Fb, 03.67.Mn

I. INTRODUCTION

In a seminal paper, Aharanov, Davidovich, and Zagury [1] introduced the discrete-time quantum walk model, which has new features when compared to the classical random walk. In special, the quantum walk spreads quadratically faster then the classical one. An alternative model was proposed by Farhi and Gutmann [2], which is called continuous-time quantum walk. Many authors have used these models to propose new quantum algorithms based on quantum walks [2, 3].

In this paper we focus our attention on quantum walks in two-dimensional lattices. Ref. [4] was one of the first to analyze 2-D quantum walks. The authors concluded that the entanglement has a negative influence on the rate of spread. Tregenna et al. [5] pointed out that this conclusion is not true in general, because it depends on the initial condition. They analyzed the full range of possible coin initial states of quantum walks starting at the origin and concluded that there are 10 types of non-equivalent coins. The Hadamard, Fourier, and Grover coins are of different types, the Grover coin being the one that produces the maximum spreading rate.

Any attempt to implement quantum walks on some physical setting faces decoherence problems. It is crucial to understand what kind of quantum walks are more resistant to decoherence. In Ref. [6] a careful analysis using non-unitary quantum operations on 1-D lattices, cycles, and hypercubes has been performed. Ref. [4] briefly analyzed decoherence effects on 2-D lattices.

The decoherence produced by broken links in 1-D lattices was analyzed in Ref. [7]. Broken-link-type decoherence is a unitary noise produced by random disruption of the links that connects neighboring sites of the lattice. This kind of noise may be relevant in implementations based on Ising spin-1/2 chains in solid-state substrates [8]. In this paper we generalize the analysis of Ref. [7] and obtain the generic evolution equation for lattices of any dimension that may be subject to this broken-link-type decoherence. Such an equation is important when analyzing quantum walks subject to many types of boundary conditions.

The main goal of the present work is to analyze the decoherence in quantum walks in 2-D lattices. We stress that some results obtained in this case cannot be obtained in the 1-D quantum walk case. More precisely, it is known that quantum coherence is disturbed by the influence of random events, which are usually modeled by some non-unitary disturbances, such as random measurements [6], or unitary disturbances, such as broken links [7]. These events are characterized by a rate defined in terms of a probability parameter p. The decoherence time goes as $1/p$, meaning that for $t \gg 1/p$ the classical behaviour emerges. Equivalently, we can say that if one increases p, the classical behavior emerges sooner. In this paper we show that this general analysis does not apply straightforwardly for non-symmetric 2-D walks. We will show that when we consider the case in which the probability of breaking links in one direction is different from the probability in the other direction, it is possible to *increase* the correlation time through an *increase* of one of these probabilities.

We have organized the paper as follows. In Section 2 we review the effect of broken links on quantum walks in 1-D lattices. The main results of quantum walks in 2-D lattices are reviewed in Section 3. In Section 4 we derive the evolution equation for 2-D quantum walks with broken links. In Section 5 we present a detailed numerical

*Electronic address: portugal@lncc.br

analysis of the decoherence produced by broken links using Hadamard, Fourier, and Grover coins, and in the last section we present our conclusions.

II. BROKEN LINKS IN 1-D QUANTUM WALKS

A coined quantum walk on an infinite line has a Hilbert space $\mathcal{H}_2 \otimes \mathcal{H}_\infty$, where $\mathcal{H}_2$ is the coin space and $\mathcal{H}_\infty$ is the line position space. The coin consists of one qubit with basis $\{|j\rangle, j \in \{0,1\}\}$. The basis for $\mathcal{H}_\infty$ is $\{|m\rangle, m\,\text{integer}\}$. The generic state of the discrete quantum walker on the infinite line is

$$|\psi(t)\rangle = \sum_{j=0}^{1} \sum_{m=-\infty}^{\infty} A_{j;\,m}(t) |j\rangle |m\rangle. \tag{1}$$

The evolution operator for one step of the walk is $U = S \circ (C \otimes I)$ where

$$C = \sum_{j,k=0}^{1} C_{jk} |j\rangle\langle k| \tag{2}$$

is the coin operator, I is the 2×2 identity matrix, and S is the shift operator given by

$$S|j\rangle|m\rangle = |j\rangle \big| m + (-1)^j \big\rangle. \tag{3}$$

As seen from this last equation, if $j = 0$ the walker moves one step to the right and if $j = 1$ the walker moves to the left, leaving the coin state unchanged. Applying the evolution operator on state (1) we obtain

$$A_{j;\,m}(t+1) = \sum_{k=0}^{1} C_{jk} A_{k;\,m-(-1)^j}(t). \tag{4}$$

Let us now analyze the evolution of the quantum walker in the case that, at time t, site m has one or both of the links connecting it to its neighboring sites broken [7]. We define the function

$$\text{link}(j;\,m) = \begin{cases} (-1)^j, & \text{if link to site } m + (-1)^j \text{ is closed,} \\ 0, & \text{if link to site } m + (-1)^j \text{ is open,} \end{cases} \tag{5}$$

where j is either 0 or 1 for the link either to the right or left of site m, respectively. Fig. 1 displays all possible cases. Note that if $\text{link}(j;\,m) = 0$ then $\text{link}(1-j;\,m+(-1)^j) = 0$.

To modify Eq. (4) in order to include the possibility of broken links, we use the following argument. Suppose that the link to the right of site m is broken, the argument being similar in the other direction. The probability flux from site m to site $m+1$ must then be diverted to site m. To calculate this flux we focus our attention on site $m+1$ and calculate $A_{0;\,m+1}(t+1)$ in terms of $A_{j;\,m}(t)$ assuming that the link is not broken. This result must be

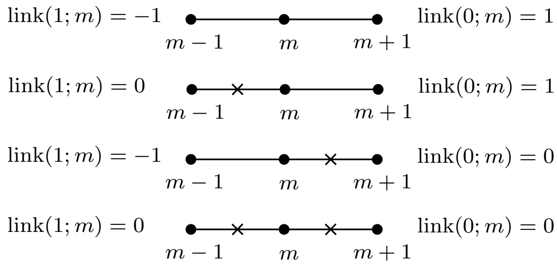


FIG. 1: Possible cases of broken links at site m and the corresponding values of $\text{link}(j;m)$.

assigned to $A_{1;\,m}(t+1)$. The formula for $A_{0;\,m}(t+1)$ does not change. Therefore, site m (instead of $m+1$) must appear in both sides of the equation for $A_{1;\,m}(t+1)$. Note also that there is a change in the line of the coin operator because $A_{1;\,m}(t+1)$ uses the line $j = 0$ of C_{jk}. The argument does not apply to $A_{1;\,m+1}(t+1)$ because it receives the flux from site $m+2$ (see [7] for more details). The above argument shows that the indices j and $m - (-1)^j$ on the right hand side of Eq. (4) must be modified yielding

$$A_{1-j;\,m}(t+1) = \sum_{k=0}^{1} C_{j+\text{link}(j;\,m),k} A_{k;\,m+\text{link}(j;\,m)}(t). \tag{6}$$

The authors of Ref. [7] have analyzed the effects that broken links produce on the quantum walk on the line. They assumed that links between neighboring sites are randomly broken with probability p per unit of time, and concluded that the evolution becomes decoherent after a characteristic time that scales as $1/p$.

III. QUANTUM WALKS IN A 2-D LATTICE

A coined quantum walk on an infinite two-dimensional lattice has a Hilbert space $\mathcal{H}_4 \otimes \mathcal{H}_\infty$, where $\mathcal{H}_4$ is the coin space and $\mathcal{H}_\infty$ is the lattice space. The coin consists of two qubits with basis $\{|j,k\rangle, j,k \in \{0,1\}\}$. We consider that the links are either along the main or along the secondary diagonals of the lattice. Thus, the basis for $\mathcal{H}_\infty$ is $\{|m,n\rangle, m,n\,\text{integers}\}$ such that $m+n$ is even.

The generic state of the quantum walker is

$$|\psi(t)\rangle = \sum_{j,k=0}^{1} \sum_{m,n=-\infty}^{\infty} A_{j,k;\,m,n}(t) |j,k\rangle |m,n\rangle. \tag{7}$$

The evolution operator for the one step of the walk is $U = S \circ (C \otimes I_4)$, where

$$C = \sum_{j,k=0}^{1} \sum_{j',k'=0}^{1} C_{j,k;\,j',k'} |j,k\rangle\langle j',k'| \tag{8}$$

is the coin operator, I_4 is the 4×4 identity matrix, and S is the shift operator given by

$$S|j,k\rangle|m,n\rangle = |j,k\rangle \big| m+(-1)^j, n+(-1)^k \big\rangle. \tag{9}$$

The walker moves along the main diagonal if the value of the coin is $|0,0\rangle$ or $|1,1\rangle$; and along the secondary diagonal if the value of the coin is $|0,1\rangle$ or $|1,0\rangle$. Note that S does not entangle the first qubit of the coin with direction n nor the second qubit with direction m. Only the combined action of the coin and shift operators can produce such entanglement.

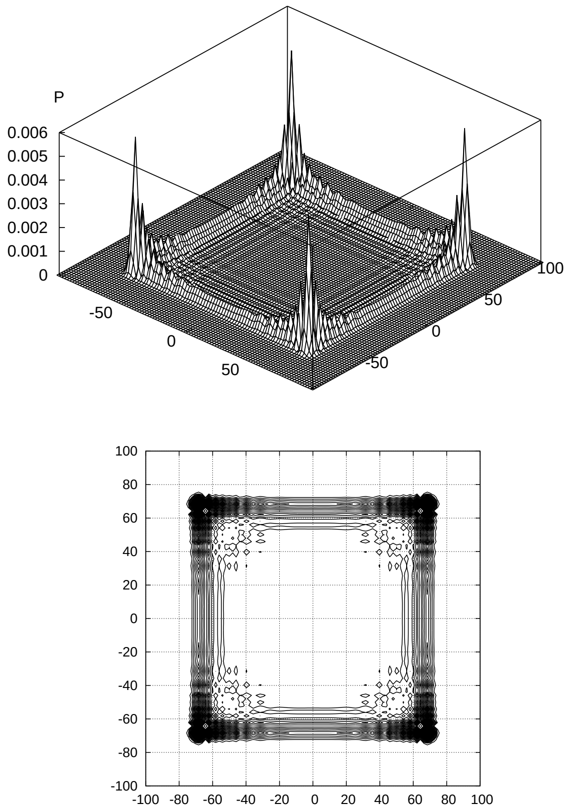


FIG. 2: The probability distribution of the Hadamard walk after 100 iterations using the initial state (13).

Applying the evolution operator on state (7) we obtain

$$A_{j,k;\,m,n}(t+1) = \sum_{j',k'=0}^{1} C_{j,k;\,j',k'} A_{j',k';\,m-(-1)^j,n-(-1)^k}(t). \quad (10)$$

The probability distribution for the walker at position $|m,n\rangle$ at time t is

$$P_{m,n}(t) = \sum_{j,k=0}^{1} |A_{j,k;\,m,n}(t)|. \quad (11)$$

Fig. 2 shows the probability distribution for the Hadamard coin ($H_4 = H \otimes H$), where $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ *i.e.*

$$H_4 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}, \quad (12)$$

after 100 steps, taking as initial state

$$|\psi(0)\rangle = \frac{1}{2}(|0\rangle - i|1\rangle)(|0\rangle + i|1\rangle)|0,0\rangle, \quad (13)$$

which produces a symmetric walk. The Hadamard coin does not entangle the coin-qubits and the shift operator does not entangle the two directions. The result is the Hadamard walk in the 2-D lattice.

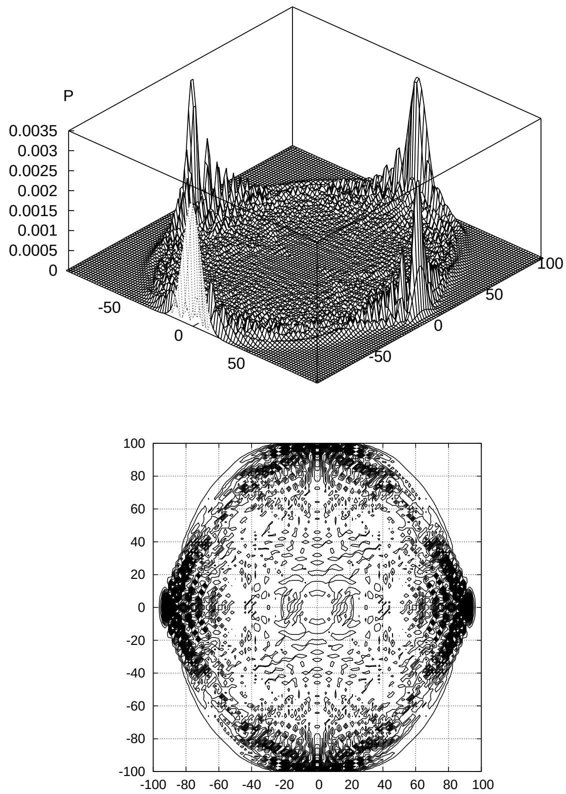


FIG. 3: The probability distribution of the Fourier walk after 100 iterations using the initial state (15).

Fig. 3 shows the probability distribution for the Fourier coin

$$F_4 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}, \quad (14)$$

after 100 steps, taking the initial state

$$|\psi(0)\rangle = \frac{1}{2}\left(|00\rangle + \frac{1-i}{\sqrt{2}}|01\rangle + |10\rangle - \frac{1-i}{\sqrt{2}}|11\rangle\right)|0,0\rangle. \quad (15)$$

Note that the density plot reveals details that are hardly seen in the 3-D plot. The walk is symmetric in the following sense: if we take any line passing through the origin, the distribution is the same in both directions. This is equivalent to saying that the plot is invariant under a rotation of π. The initial state (15) was chosen to guarantee a maximum of spreading when the walk starts at the origin [5]. This is the most interesting situation in decoherence analysis.

In Fig. 4 we show the probability distribution for the Grover coin

$$G_4 = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}, \tag{16}$$

after 100 steps, taking the initial state

$$|\psi(0)\rangle = \frac{1}{2}(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)|0,0\rangle. \tag{17}$$

The density plot illustrates the remarkable properties of the Grover coin for the initial state (17). The walk is highly symmetric, since it is apparently invariant under a rotation of $\pi/2$. The walk is delocalized, having a rather empty central region of about 1/3 of the reachable radius which has an almost zero probability distribution.

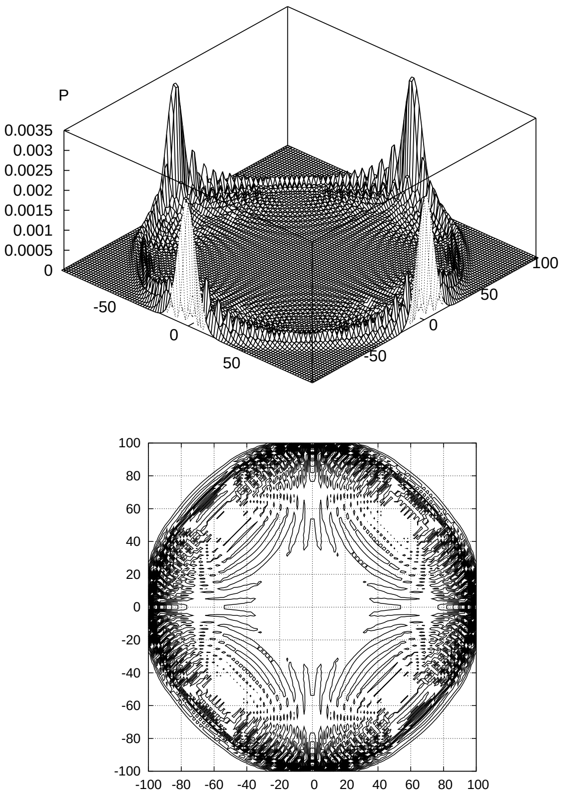


FIG. 4: The probability distribution of the Grover walk after 100 iterations using the initial state (17).

Other relevant references regarding 2-D and higher dimensional quantum walks are [9, 10].

IV. BROKEN LINKS IN 2-D QUANTUM WALKS

The argument used in Sec. II to derive the equation for the evolution of the amplitudes (6) in the one-dimensional case can be easily generalized to the case of two-dimensional walks with broken links. Two functions are now required to specify the broken links, one for each direction,

$$\mathrm{link}_1(j,k;\,m,n) = \begin{cases} (-1)^j, & \text{if link to site } m+(-1)^j, \\ & \quad n+(-1)^k \text{ is closed,} \\ 0, & \text{if the link is open,} \end{cases} \tag{18}$$

$$\mathrm{link}_2(j,k;\,m,n) = \begin{cases} (-1)^k, & \text{if link to site } m+(-1)^j, \\ & \quad n+(-1)^k \text{ is closed,} \\ 0, & \text{if the link is open,} \end{cases} \tag{19}$$

where $j,k \in \{0,1\}$. The equation that generalizes (10) is

$$A_{1-j,1-k;\,m,n}(t+1) = \sum_{j',\,k'=0}^{1} C_{j+\mathrm{link}_1(j,k;\,m,n),k+\mathrm{link}_2(j,k;\,m,n);\,j',k'} A_{j',k';m+\mathrm{link}_1(j,k;\,m,n),n+\mathrm{link}_2(j,k;\,m,n)}(t) \tag{20}$$

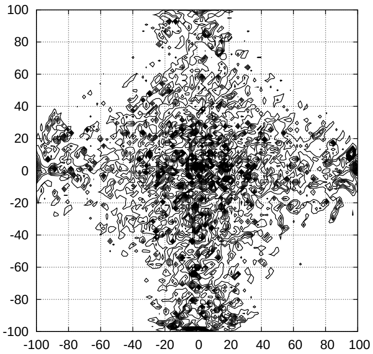
We easily see that the above equation reduces to (10) if there are no broken links. When implementing this equation, one must impose that $\mathrm{link}_1(1-j,1-k;\,m+(-1)^j,n+(-1)^k) = 0$ if $\mathrm{link}_1(j,k;\,m,n) = 0$, and similarly with link_2.

The evolution equation for quantum walks in n-dimensional lattices is a generalization of Eqs. (18), (19), and (20). In this case one needs to use n link functions defined analogously to Eqs. (18) and (19). Eq. (20) must be modified accordingly, adding each link function to its corresponding index. With these equations in hand, it is possible not only to analyze broken-link-type decoherences in n-dimensional lattices, but also to analyze the decoherence-free walks in lattices with reflecting boundary conditions. In fact, one can choose a variety of lattice topologies by permanently breaking the relevant links.

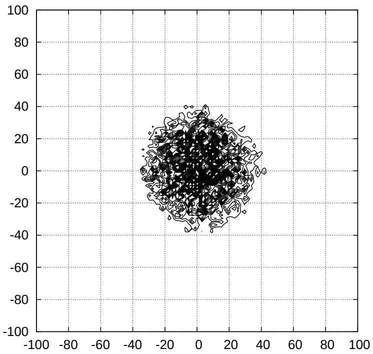
V. RESULTS AND DISCUSSION

Now we analyze numerically the decoherence effects of broken links in the 2-D walk described in the previous sections. We give more attention to results that differ from those known in the 1-D case.

Fig. 5(a) shows the decoherence effects in the Grover walk when $t \approx 1/p$, where p is the probability of breaking the links, in the case $p = 0.01$, $t = 100$. Both quantum and classical behaviors are present, but the classical one starts to take over the quantum behaviour. Compare



(a) Probability distribution for $p = 0.01$. The classical behaviour starts to dominate over the quantum behaviour at $t \approx 1/p$.



(b) Probability distribution for $p = 0.1$. The classical behaviour has emerged since $t \gg 1/p$.

FIG. 5: Decoherence of the Grover walk at $t = 100$ with initial state (17).

to Fig. 4, for $p = 0$, which is delocalized at the origin. The standard deviation of the probability distribution in Fig. 5(a) is still larger than in the classical case. The situation changes dramatically when $t \gg 1/p$, as illustrated in Fig. 5(b) for $p = 0.1$ and $t = 100$. In this case the classical behaviour is fully developed. It is easy to show that the standard deviation grows, in this case, as $\sqrt{t}$.

The study of the transition from the quantum to the classical behavior is made easier by plotting the evolution of the standard deviation (σ). Fig. 6 shows σ for the Grover walk for many values of p in a log-log scale. The continuous lines represent the quantum and classical standard deviations when there are no broken links ($p = 0$). The quantum curve for $p = 0$ has inclination 1 for $t > 2$. When $p > 0$, all curves have a similar behavior: they have a slope 1 for $t \ll t_d$, which gradually decreases to $1/2$ for $t \gg t_d$, where t_d is the decoherence time, which is usually approximated by $1/p$, but which we discuss below.

The decoherence time can be estimated in the following way. For small values of the evolution time t, the number of broken links inside the area accessible to the walker is still small. This number increases with time, as the boundary of the accessible region expands. As the

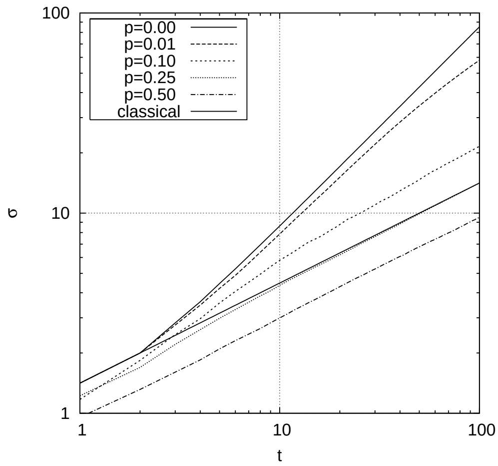


FIG. 6: Evolution of the standard deviation of the Grover walk with broken links for an initial state (17), in the cases $p = 0.01, 0.1, 0.25$, and 0.5. The continuous lines are the quantum and classical walks with no broken links.

Hadamard walk spans an area $2t^2$ at time t, the cumulative number of broken links after a time T is

$$\sum_{t=0}^{T} 2\,t^2 p = \frac{1}{3}\,T\,(T+1)\,(2T+1)\,p.$$

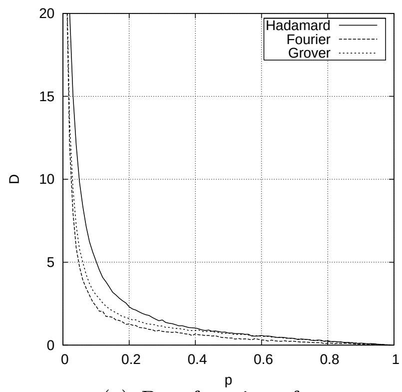
It is natural to approximate t_d by the time at which the cumulative number of broken links equals the total number of links in the accessible area of the walker, which is $2T^2$. Then we get $t_d \approx 3/p$, for $p \ll 1$.

There are several novel results in the case of 2-D walks when compared to 1-D quantum walks. First of all, while all 1-D walks starting at the origin can be obtained from the Hadamard walk [11–13], 2-D walks have many non-equivalent coins. In fact, the standard deviation of the Hadamard, Fourier, and Grover walks are different, when the walker starts at the origin with initial states (13), (15), and (17), respectively, which give the maximum spreading rates for each case. We still have, in all cases, $\sigma = \alpha t$, but the values of α vary from coin to coin. Numerically we find, in the case $p = 0$, $\alpha_H = 0.77$, $\alpha_F = 0.80$, and $\alpha_G = 0.85$, respectively. We notice that the Grover walk leads to the largest diffusion rate among all the coins considered [5].

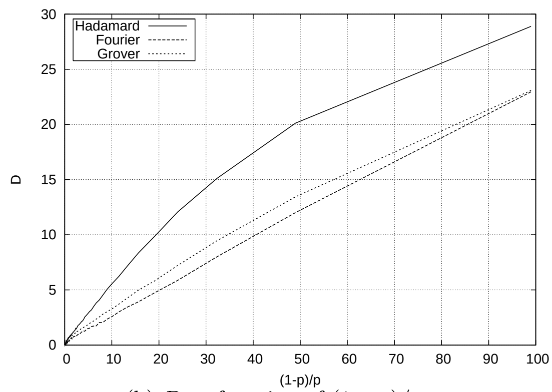
The diffusion coefficient is defined by

$$D = \frac{1}{2}\lim_{t\to\infty}\frac{\partial \sigma^2}{\partial t}. \qquad (21)$$

Figs. 7(a) and 7(b) show that the coins have different sensitivity to decoherence. When $p = 0$, $D = \infty$ since σ^2 has a quadratic increase. When $p > 0$, D is finite as, when $t \gg t_d$, σ^2 has a linear increase. The diffusion coefficient D measures half of the inclination of the asymptotic line in a σ^2 *vs.* t log-log plot. From Fig. 7(b) we conclude that the Hadamard coin leads to a quantum walk more resistant to decoherence than the Grover coin, and also that Grover is more resistant than Fourier. The difference between the Fourier and Grover walkers with broken links is small when compared to the Hadamard case. When p is very close to 1 the coins are similar. This is because, in this case, the noise is so intense that the diffusion rate is even below that for a classical random walk. Certainly this case has little, if any, practical interest. We remark that the preceding analysis uses the initial states that produce the largest diffusion rate for each coin, and not the same initial state for all cases.



(a) D as function of p.



(b) D as function of $(1-p)/p$.

FIG. 7: The diffusion coefficient D for quantum walkers moving on a lattice with a broken link probability parameter p, as function of the coin operator employed.

From the plot of Fig. 7(a), it is also possible to estimate the noise level for which the diffusion rate of the walk equals the classical diffusion rate with no broken links, *i.e.* $D = 1$. We found the approximate values: $p_H = 0.41$ (Hadamard), $p_F = 0.34$ (Fourier), and $p_G = 0.25$ (Grover). For values of p above p_H, p_G, p_G, the frequency of broken links is so high that the walker spreads more slowly than a classical random walk with no broken links. Note also that p_H is very close to the corresponding value for the 1-D walk found in ref. [7]. This was expected since a 2-D Hadamard walk corresponds to two independent 1-D Hadamard walks. This analysis does not mean that for p greater than the above values, there are no quantum correlations. Such correlations persists as long as $t < t_d$ for all values of p, although one should also note that the decoherence time t_d becomes quite small when p approaches 1.

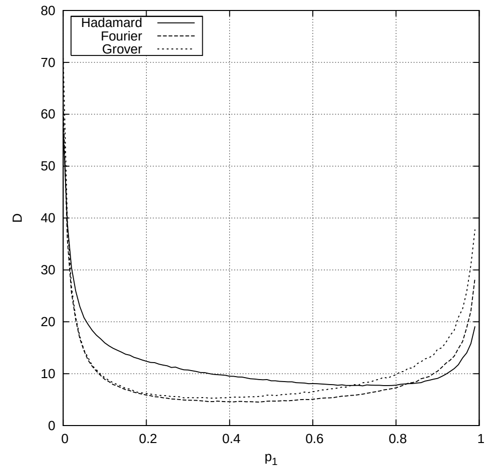


FIG. 8: Diffusion coefficient as function of p_1 for $p_0 = 0$ (no broken links along parallels to the main diagonal). The minimum value of the diffusion coefficient (maximum decoherence) is obtained, in all cases, for a value of $p_1 < 1$. See text for additional details.

Thus far we have described the decoherence produced by broken links with equal probabilities for both directions. It will be interesting to study the non-symmetric case, this is, when the probability to break links along parallels to the main diagonal (p_0) is different from the probability in the perpendicular direction (p_1). One would expect that the decoherence always increases when either of the probability parameters associated with broken links along the two diagonals increases. However, in the non-symmetric case, a quite remarkable situation takes place. Let us consider the diffusion coefficient in the case where there are no broken links along the main diagonal direction, $p_0 = 0$, as function of p_1. This is illustrated, for the three different coins considered, in Fig. 8. We note that D has a minimum value for, approximately, $p_1 = 0.72$ (Hadamard), $p_1 = 0.47$ (Fourier), and $p_1 = 0.35$ (Grover), and thus it increases when p_1 approaches 1. This result should be compared to that of Fig. 7 for $p_0 = p_1$, in which case the diffusion coefficient goes to 0 as p_0 and p_1 go simultaneously to 1.

The Figs. 9 and 10 help understand what is physically taking place. The case when $p_0 = 0$ and $p_1 \approx 1$ is similar to a 1-D quantum walker that has a probability 1/2 to move along the main diagonal (in either direction) and 1/2 to keep still. This walk is described by a shift oper-

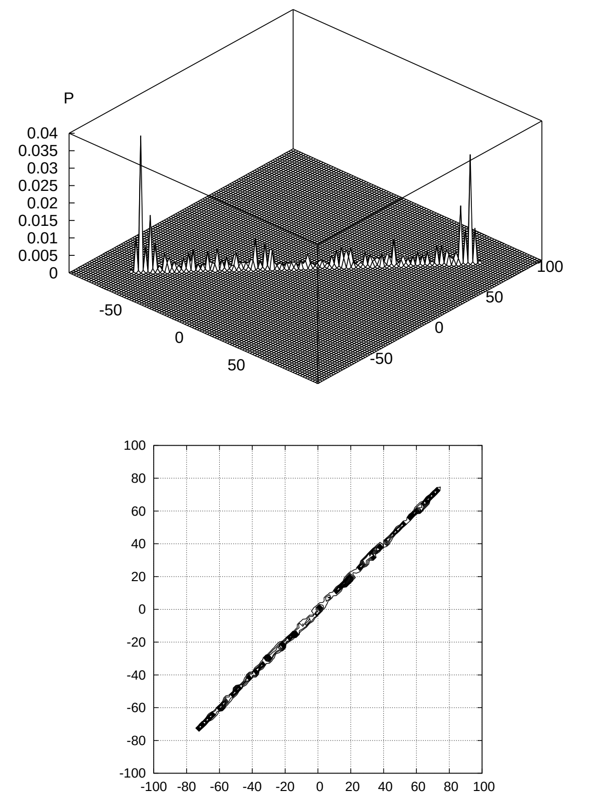


FIG. 9: Non-symmetric Grover walk for $p_0 = 0$ and $p_1 = 0.99$, at time $t = 100$, for the initial state (17). The large value of p_1 forces the walker to stay very close to the main diagonal.

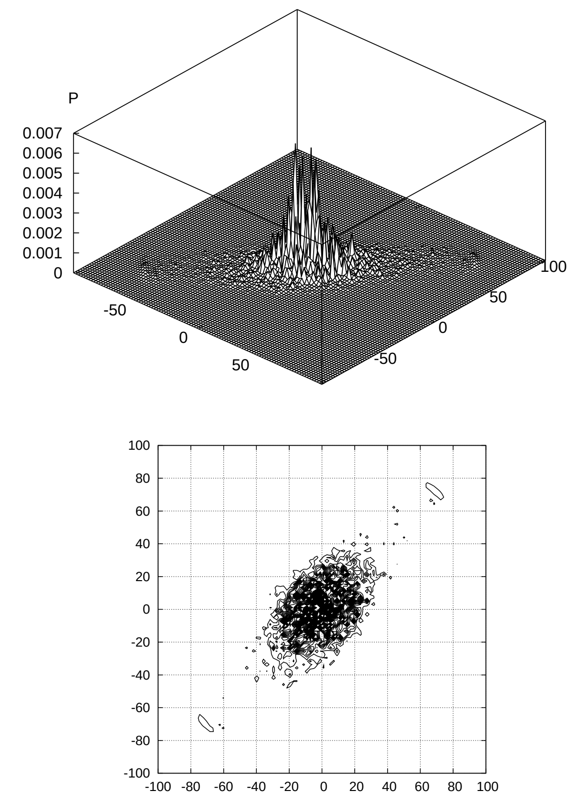


FIG. 10: Similar to Fig. 9 for $p_1 = 0.35$. The walker partially spreads more along the secondary diagonal, losing its coherence faster than in the previous case.

ator of the form

$$S = |00\rangle\langle 00| \otimes \sum_{m=-\infty}^{\infty} |m+1\rangle\langle m| + \\ |01\rangle\langle 01| \otimes \sum_{m=-\infty}^{\infty} |m\rangle\langle m| + \\ |10\rangle\langle 10| \otimes \sum_{m=-\infty}^{\infty} |m\rangle\langle m| + \\ |11\rangle\langle 11| \otimes \sum_{m=-\infty}^{\infty} |m-1\rangle\langle m|. \tag{22}$$

This variation of the 1-D quantum walker was studied in Ref. [14]. The probability distribution for the walk using the shift operator (22) and the Grover coin (16) is depicted in Fig. 11. Note that the qualitative behavior of this walker is very similar to the one shown in the left panel of Fig. 9. Analyses performed with Hadamard and Fourier coins have resulted in similar matches.

For $p_1 \approx 0.35$, the walker partially spreads along the secondary diagonal direction while losing coherence, as it can observed in Fig. 10. The relation $t_d \approx 1/p$ does not apply in the non-symmetric case since we have two probabilities to consider. Fig. 10 shows that the decoherence time along the secondary diagonal is smaller than

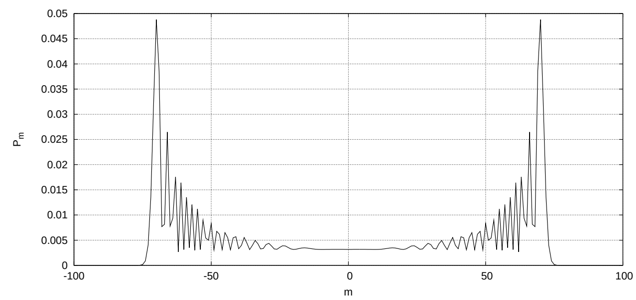


FIG. 11: Probability distribution of the 1-D quantum walk with shift operator (22) using Grover coin and initial condition $|-\rangle|-\rangle|0\rangle$, where $|-\rangle = (|0\rangle - |1\rangle)/2$.

along the main diagonal. Correlations still persist along the main, but are completely lost along the secondary diagonal. As a first approximation, one could associate a decoherence time to each direction: $t_d^{(0)} \approx 1/p_0$ and $t_d^{(1)} \approx 1/p_1$.

VI. CONCLUSIONS

We have analyzed the decoherence produced by randomly breaking links in a 2-D lattice. We have used the

Hadamard, Fourier, and Grover coins, taking as initial condition the one that leads to a maximum rate of spread. We have found that the Hadamard walk is more resistant to this type of decoherences than the Grover walk, which, in turn, is more resistant than the Fourier walk. We have also obtained the evolution equation for quantum walks in n-dimensional lattices with broken links.

These extensions to higher dimensional lattices open the way for several studies. We have seen how the difference in breaking probability along two orthogonal directions lead to a transition first from a coherent 2-D walk to a decoherent 2-D walk, and then to a coherent 1-D one. Such studies may be easily carried over to three dimensions.

The treatment presented in this work allows also to study the evolution of quantum walkers on lattice regions of arbitrary shape, through the procedure of permanently breaking the appropriate links in order to define its boundary. A possible application of this method is to study the transmission of quantum walkers through open billiards [15], or in a region where the corresponding classical motion would be chaotic. Other applications that could be considered are the problem of quantum percolation, and the propagation of the walkers in inhomogeneous regions, such as the interface of two regions with different conductivities. Work along these lines is in progress.

Acknowledgments

We thank G. Abal for useful discussions. This work was funded by FAPERJ and CNPq.

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