

Quantum models of the mind: Are they compatible with environment decoherence?

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Quantum models of the mind associate consciousness with coherent superposition of states in the brain. Some authors consider consciousness to be the result of a kind of internal quantum measurement process in the brain. In this paper we discuss the ideas of Hameroff-Penrose and Tegmark and their calculation for an estimate of decoherence time. We criticize the Hameroff-Penrose model in the context of a quantum brain model by gravitational collapse orchestrated objective reduction (OOR), assumed by Penrose, and we propose instead that the decoherence process is caused by interaction with the environment. We consider it useful to exploit this possibility because of the growing importance of the decoherence theory in quantum measurement, and also because quantum mechanics can be applied to brain study independently of the Hameroff-Penrose model for mind and consciousness. Our conclusion is that the Hameroff-Penrose model is not compatible with decoherence, but nevertheless the quantum brain can still be considered if we replace gravitational collapse OOR with decoherence. However, our result does not agree with Tegmark's conclusion of refuting not only the Hameroff-Penrose gravitational collapse but also the quantum brain, based on decoherence time calculations in specific cases in the brain. In spite of this fact we also disagree with some points of the response to Tegmark's article given by Hagan, Hameroff, and Tuszynski.

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I. INTRODUCTION

Different works on the brain and mind problems have used quantum theory to explain the emergence of consciousness [1–8]. There are, in quantum theory as well as in statistical physics, collective phenomena irreducible to individual components of the system. The conjecture is that collective quantum phenomena produce coherent states in the brain. As we shall see in this paper, decoherence was not taken into account generally in current quantum models of the mind until very recent polemical works [1,9].

Hameroff and Penrose [2,3] formulated what they call orchestrated objective reduction (OOR) in the brain, by extending the objective interpretation of quantum measurement as a gravitational physical mechanism, suggested by Penrose [5] instead of decoherence. In this paper we discuss the Hameroff-Penrose model [2,3] and Tegmark's arguments [9] against the OOR. We consider the possibility of substituting the idea of gravitational collapse (OOR) by the natural decoherence process to produce consciousness. We concentrate our discussion on quantum aspects of the brain. Hameroff and Penrose assume a quantum computing process in microtubules, which constitute the skeleton of neural cells. We show a mistake concerning the temperature regime of the decoherence process which was considered by Hameroff and Tuszynski [1] in their criticism of Tegmark's calculation of decoherence rate. In Sec. II we present a formal view of the constitution of the microtubules in the brain, as well as of the Hameroff-Penrose quantum model of consciousness production. In Sec. III we point out some problems of this model and we discuss Tegmark's critical paper [9]. Our calculation

of decoherence time in the brain system, by assuming ion and dipole interactions, are in Sec. IV. The conclusion and final comments are presented in Sec. V.

II. MICROTUBULES AND QUANTUM PROCESS

Microtubules are hollow cylinders comprised of an exterior surface with a cross-section diameter of 25 nm with 13 arrays of protein dimers called tubulines. The interior of the cylinder contains ordered water molecules, which implies the existence of an electric dipole moment and an electric field [10–14]. The microtubules comprise the internal scaffolding (the cytoskeleton) in all cells including neurons. Cytoskeletal structures are determinant of both structure and function and are dynamically active, performing activities which are instrumental to cellular organization [10–14]. See Fig. 1.

The walls of microtubules in the cytoskeleton of neurons can work as cellular automata, able to store information [15] and to make computation by using combinations of the two possible states (as dimers) of the tubulins that constitute these walls [1,11–14]. The interior of the microtubule works as an electromagnetic wave guide, full of water in an organized collective state, able to transmit information through

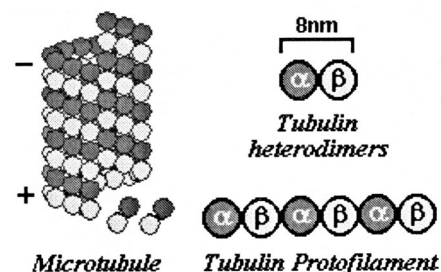


FIG. 1. Illustration of microtubule, tubulin, and protofilament.

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the brain. A gelatinous state of water in brain cells, which was observed by Watterson [16], favors a macroscopic collective effect of the coherent quantum state at a microscopic level. The conjugation of these possibilities can result in a coherent superposition of quantum states, embracing a very large number of microtubules.

Consciousness in the Hameroff-Penrose quantum model, using the microtubules structure, is produced in two steps. While the coherent superposition expands, there is a quantum computation through tubulins, such as dimers, which can work as cellular automata in the walls of microtubules [15], and a propagation of information by wave guides inside microtubules of neurons. In this part of the cycle, in which the coherent superposition of quantum states persists, there is a preconsciousness state to the person. The second step is just the orchestrated reduction which produces consciousness [1–5].

III. QUANTUM BRAIN AND DECOHERENCE

A possibly vulnerable point in the quantum models of consciousness is the interaction between system and environment. According to the decoherence theory, macroscopic objects obey quantum mechanics. The interaction with the environment in this theory causes decoherence, which destroys quantum effects of macroscopic objects [17–19]. For this reason, these objects behave according to classical physics and Newton mechanics in an approximation of quantum theory. But for us this natural time limit for quantum superposition, generated by decoherence, is the fundamental process that yields consciousness. If we conciliate decoherence effects and the biological theory of Fröhlich [10,20], we can construct an interesting model to quantum mind-brain process. The Fröhlich theory describes quantum superposition for biomacromolecules by considering the polarization and vibration properties of these molecules. Vibrations in polarized systems generate electromagnetic fields which can mediate interactions among the units of the system. The nonlinearity of his model induces an energy transference among the vibration modes. This process can provide a condensation of the energy and consequently coherence in living systems [10,20].

The questions to be answered are: Can we deny the hypothesis of gravitational effect and maintain Hameroff and Penrose's model to explain consciousness? In this hypothesis, can we use this model with decoherence? If we answer "yes" to the above questions we invalidate Penrose's hypothesis of nonalgorithmic theory which explains the emergent properties of the mind. In order to make a comparison with the predictions of Hameroff and Penrose, we discuss the decoherence time in the brain calculated by Tegmark [9], whose conclusion was to discard the relevance of the quantum approach to the brain. The period of time in which coherence is sustained in the brain was estimated at 500 ms by Hameroff and Penrose [2,3] based on the biological time of a brain response to external stimuli. This period is many times greater than the time of decoherence, considered an extremely efficient and fast natural process. The time of reduction by gravitational effect, calculated by Penrose, for water

drops 10^{-4} cm and 10^{-3} cm in diameter is, respectively, 0.1 s and 10^{-6} s. But typical decoherence time is estimated to be much shorter. They are 10^{-23} s and 10^{-9} s, respectively, for systems with a radius of 10^{-3} cm and 10^{-5} cm [18,22]. Hameroff and Penrose [3] have calculated the number of tubulins under coherent superposition assuming a coherence time t based on the typical response time of the brain to external stimulus. The corresponding critical energy for OOR is related to time t by the quantum uncertainty relation

$$E = \frac{\hbar}{t}. \quad (1)$$

The numerator is the Planck constant ($6.6260755 \times 10^{-34}$ J s) over 2π and the denominator is the time t which gives the order of magnitude. The critical energy E is given by the gravitational self-energy.

IV. COMPARISONS OF THE DECOHERENCE RATES

We assume, in place of gravitational collapse, that coherence grows as the system is isolated by some mechanism until the coherent superposition reaches a critical situation, at which point decoherence is activated. When the limit given by the critical value of some physical parameter is reached, interaction with the environment becomes sufficient to cause decoherence and, therefore, to destroy the coherent collective state. The critical size has no connection to the space-time structure. Therefore, the quantum system remains protected against decoherence by interaction with the environment, as isolated systems up to reach the critical size.

We can make a first rough estimate of t based on a macroscopic thermodynamic prescription of the critical energy, used in place of gravitational autointeraction in the Hameroff and Penrose [2,3] model. However, we can examine the problem from another point of view by making a microscopic quantum description. Tegmark [9] has taken into account, in a detailed model, the relevant degrees of freedom in the brain. His conclusion is that the system is not sufficiently shielded from environmental influence to maintain quantum coherence in such a way as to allow quantum computation. He considers as limits two scales: a macroscopic one for superposition of neurons firing and a microscopic one for microtubules. In the first one he estimates $t = 10^{-20}$ s and in the second one $t = 10^{-13}$ s. To calculate the latter value Tegmark considers the propagation along the microtubule, in the direction of its axis, z , of a kinklike perturbation which changes the tubulin electric dipole from $+p_0$ to $-p_0$:

$$p(z) = \begin{cases} +p_0, & z \gg z_0 \\ -p_0, & z \ll z_0, \end{cases} \quad (2)$$

For this calculation the author uses the total charge generated by positive ions of calcium in a ring of tubulins at the cross section of the microtubule, at the point of the kinklike propagation. This charge, distributed around the microtubule wall at the point of the perturbation, interacts with distant ions of the environment through the Coulomb potential. The distributed charge is treated as one particle and the environmental

ion acts as the other particle. Several approximations are made. The Coulomb potential R^{-1} is expanded in a Taylor series around R up to the second order term. The decoherence effect is produced by the term of the Taylor expansion with a second order derivative of the potential, which is proportional to R^{-3} , as one observes in the tidal force exerted by the moon's gravitational attraction.

The formula used by Tegmark for the decoherence time of microtubule quantum states generated by interaction with environmental ions is

$$\tau = \frac{R^3 \sqrt{M \kappa T}}{K N e^2 s}, \quad (3)$$

where K is the Coulomb constant, R is the distance between the interacting microscopic system in the tubulin and the environmental ion, M is the mass of the ion, N is the number of elementary charges in the microtubule interacting system, s is the maximal "separation" between the position of two kinks in quantum superposition, e is the electron charge, κ is the Boltzmann constant, and T is the temperature. If we consider the values of the physical constants $K = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$, $\kappa = 1.38 \times 10^{-23} \text{ J K}^{-1}$, and $e = 1.6 \times 10^{-19} \text{ C}$ and taking $T = 309 \text{ K}$, M the mass of a water ion, $18 \times$ the *proton mass* ($1.67 \times 10^{-27} \text{ kg}$), $N = 10^3$ estimated by Tegmark, who assumed R and s with the order of magnitude of the microtubule diameter $24 \times 10^{-9} \text{ m}$, we have from formula (3) the order of magnitude $t = 10^{-13} \text{ s}$.

Hagan, Hameroff, and Tuszynski [1] in their response to Tegmark's paper agreed with his result at the level of neuron, in the macroscopic scale, but not in the microscopic scale where tubulins and microtubules play a role. One reason is that Tegmark's model takes kinklike soliton waves [21] along the microtubule, while OOR considers the superposition of different conformational states of a tubulin dimer [1]. In spite of their criticism, Hagan, Hameroff, and Tuszynski have used the Tegmark model with some modifications, using electric dipole momentum in place of the Coulomb potential to describe tubulin interaction with an environmental ion:

$$\tau = \frac{R^4 \sqrt{M \kappa T}}{3 K e p s} \Omega_{dipole}, \quad (4)$$

where R is the distance between the environmental ion and the dipole generated by the tubulin of the microtubule and p is the electric dipole momentum; the other variables are analogous to those of formula (3), unless the last factor in (4), which is a geometric factor involving the angles between the directions of the line from the dipole to the ion, of the dipole, and of the separation. It is assumed to be of order of 1 [1]. In their approximation the authors claim that the decoherence time lies in the range $10^{-6} \text{ s} - 10^{-4} \text{ s}$.

However, if we apply formula (4) with R , M and s having the same order of magnitude of the values used by Tegmark, but using for the tubulin dipole momentum [1] and considering the direction of the microtubule axis the value $p = 10^{-27} \text{ C m}$, we go back to our previous result, $t = 10^{-10} \text{ s}$.

V. RESULTS

The authors have also criticized formula (3) used by Tegmark [9] because it predicts that the decoherence time increases with the square root of the temperature. We can see from very general formulas (1) and (2) that time t decreases when temperature T increases. On the contrary, according to formula (3) of Tegmark at low temperature, where the interaction with the environment must be minimal, the decoherence is faster, in contradiction with experience. Coherent macroscopic states, such as superconductivity, happen at temperature approaching zero. We show in what follows that Tegmark's formula is not valid at low temperature. This is not the case of the human body but it shows that Tegmark's estimates cannot be used generally. That is, the formulas (3) and (4) are just an approximation for a right regime of temperature.

To discuss Tegmark formula (3), let us consider the density matrix for the positions of the two particles 1 (charged macromolecule) and 2 (environmental ion) interacting through a potential $V(r)$. From the Schrödinger equation for density matrix ρ the system evolves as

$$\frac{d\rho}{dt}(r, t) = [H, \rho(r, t)], \quad (5)$$

where H is the Hamiltonian of system. In this case we assume $H = V(r)$; therefore, we have

$$\rho(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2, t) = \rho(0) \exp\left[-\frac{i}{\hbar} [V(\vec{R}) - V(\vec{R}')]t\right], \quad (6)$$

where

$$\begin{aligned} \vec{R} &= \vec{r}_2 - \vec{r}_1, \\ \vec{R}' &= \vec{r}'_2 - \vec{r}'_1. \end{aligned} \quad (7)$$

Decoherence drops the nondiagonal part of the macroparticle 1 reduced density operator due to the interaction with particle 2; when we assume that systems 1 and 2 start in a tensorial product state $\rho = \rho_1 \rho_2$, the density operator is averaged by tracing over the environment. The result is

$$\begin{aligned} \rho_1(\vec{r}_1, \vec{r}_2, t) &= \text{Tr}[\rho(\vec{r}_1, \vec{r}_2, \vec{r}'_1, \vec{r}'_2, t)] \\ &= \rho_1(\vec{r}_1, \vec{r}_2, t) \exp(-\Lambda t) \quad \text{for } \vec{r}_1 \neq \vec{r}'_1. \end{aligned} \quad (8)$$

That is, the nondiagonal part vanishes for $t > \Lambda^{-1}$. To obtain this result, we choose

$$\begin{aligned} \vec{r}_1 &= (0, 0, 0), \\ \vec{r}'_1 &= (x_1, y_1, z_1), \\ \vec{r}_2 = \vec{r}'_2 &= (x_2, d, 0). \end{aligned} \quad (9)$$

The first two relations mean a coordinate axis choice, while the last one means an approximation where the environmental ion (particle 2) moves along the x axis (d is a constant)

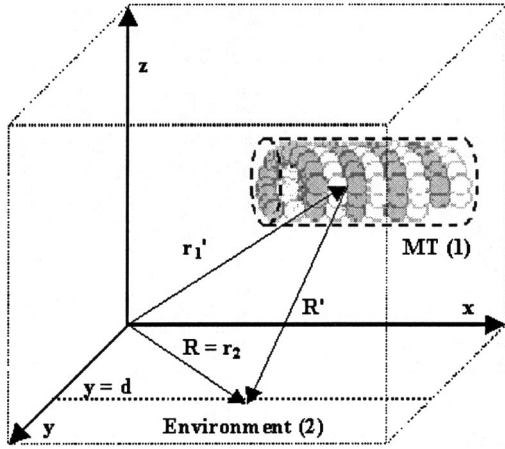


FIG. 2. Schematic illustration of the coordinates of the environment-MT system.

and the change of direction of its motion is neglected in the interaction, assumed as too small. The shortest distance from it to the particle 1 is d . So

$$\begin{aligned}\vec{R} &= \vec{r}_2, \\ \vec{R}' &= \vec{r}_2 - \vec{r}_1'.\end{aligned}\quad (10)$$

We can expand the Coulomb potential up to the second order term,

$$V(\vec{R}') = Kq_1q_2 \frac{1}{|\vec{r}_2 - \vec{r}_1|} \cong Kq_1q_2 \left(\frac{1}{|\vec{r}_2|} + \frac{x_1x_2 + y_1d}{|\vec{r}_2|^3} \right) \quad (11)$$

and hence

$$V(\vec{R}') - V(\vec{R}) = Kq_1q_2 \left(\frac{x_1x_2 + y_1d}{|\vec{r}_2|^3} \right), \quad (12)$$

where q_1 and q_2 are the charges of particles 1 and 2.

Figure 2 shows the relation MT-environment expressed in Eq. (10).

The next step is to assume a separable form for the density matrix and a Gaussian distribution with zero mean and variance $\lambda\sqrt{2}$ for particle 2,

$$\begin{aligned}\rho_1(\vec{r}_1, \vec{r}_2, t) &= \frac{\rho_1(0)}{\sqrt{4\pi\lambda}} \int \exp\left[-\left(\frac{x_2}{2\lambda}\right)^2\right] \\ &\times \exp\left[-\frac{Kq_1q_2t}{i\hbar} \left(\frac{x_1x_2 + y_1d}{(x_2^2 + d^2)^{3/2}}\right)\right] dx_2.\end{aligned}\quad (13)$$

For $d \gg x_2$ we can approximate $(x_2^2 + d^2)^{3/2} \cong d^3$ and with an appropriate algebraic manipulation Eq. (13) becomes

$$\rho_1 = C \exp\left[\frac{ty_1Kq_1q_2}{i\hbar d^2} - \left(\frac{tx_1\lambda Kq_1q_2}{\hbar d^3}\right)^2\right] \Phi, \quad (14)$$

where

$$\begin{aligned}\Phi &= \int_{-\infty}^{+\infty} \exp\left[-\left(\frac{x_2}{2\lambda} + \frac{itx_1\lambda Kq_1q_2}{\hbar d^3}\right)^2\right] dx_2, \\ C &= \frac{\rho_1(0)}{\sqrt{4\pi\lambda}}.\end{aligned}$$

The last integration in Eq. (14) yields a constant number; the first imaginary term in the exponential is a phase factor, while the second one produces a decrease of the nondiagonal part of the density matrix with a characteristic decoherence time,

$$\tau = \frac{\hbar d^3}{x_1\lambda Kq_1q_2}. \quad (15)$$

If we assume thermal constraint $(\Delta p)^2/M \approx \kappa T$ and take the uncertainty principle $\Delta x \approx \hbar/(M\kappa T)^{1/2}$ as Tegmark did [4], we rescue formula (3).

The square root of κT in the numerator of Eq. (15) depends on the approximation used $\lambda \leq d$ and $x_2 \leq d$. However, if we use a much broader function to represent the environmental ion in the density matrix, such that it vanishes effectively for $x_2 = \lambda \gg d$, the situation is changed in the expression (13)

$$\rho_1(\vec{r}_1, \vec{r}_2, t) = C \int \exp\left[-\frac{1}{4} - \frac{Kq_1q_2t}{i\hbar} \frac{x_1\lambda + y_1d}{(\lambda^2 + d^2)^{3/2}}\right] dx_2. \quad (16)$$

The integral above will vanish if the imaginary exponential oscillates upwards too close to the limit, producing a cancellation between positive and negative parts of the function in the integration, which vanishes when

$$t \gg \frac{\hbar}{Kq_1q_2} \frac{(\lambda^2 + d^2)^{3/2}}{x_1\lambda + y_1d} \cong \frac{\hbar}{Kq_1q_2} \frac{\lambda^2}{x_1}. \quad (17)$$

By simple inspection we see that, with $\lambda = \hbar/(M\kappa T)^{1/2}$, the factor κT goes down to the denominator in Eq. (17), as usual, differently from the Tegmark formula (3),

$$\tau = \frac{1}{Kq_1q_2} \frac{\hbar^3}{x_1 M \kappa T}. \quad (18)$$

At very low temperature τ becomes too high and the approximation used in formula (3) does not work. In this case we must use formula (18) instead. So we remove the obstacle put by Hagan, Hameroff, and Tuszynski [1] against the validity of the Tegmark formula.

For the electric dipole case (4) we can use the same process used previously. So, let us consider a composite system of two particles, where particle 1 is the electric dipole and particle 2 represents the environment. Both of them interact through a dipole potential $V(r)$ which expanded,

TABLE I. Calculated values of the decoherence time scale.

Quantum superposition of	Decoherence time calculated
Superposition of neural firing [4]	10^{-20} s
Soliton superposition [4]	10^{-13} s
OOR superpositions [1]	10^{-5} – 10^{-4} s
Decoherence model (MT-ion interaction)	10^{-9} s
Decoherence model (MT-dipole interaction)	10^{-16} s

$$V(\vec{R}') = Kq \frac{\vec{p} \cdot (\vec{r}_2 - \vec{r}'_1)}{|\vec{r}_2 - \vec{r}'_1|^3} \cong Kq \left(\frac{\vec{p} \cdot \vec{r}_2}{|\vec{r}|^3} + \frac{3\vec{p} \cdot \vec{r}_2 |\vec{r}'_1|}{|\vec{r}_2|^4} \right). \quad (19)$$

From Eq. (5) and taking all previous considerations, the density matrix of system becomes

$$\rho_1(\vec{r}_1, \vec{r}_2, t) = C \int \exp \left[- \left(\frac{x_2}{2\lambda} \right)^2 - \frac{3|\vec{r}'_1|Kq}{i\hbar} \times \left(\frac{p_x x_2 + p_y d}{(x_2^2 + d^2)^2} \right) t \right] dx_2. \quad (20)$$

For a regime where $d \gg x_2$ we can approximate $(x_2^2 + d^2)^2 \cong d^4$ and therefore

$$\tau = \frac{d^4 \sqrt{M \kappa T}}{3Kqps} \Omega_{dipole}, \quad (21)$$

where we assume $p_x = p \cos(\alpha)$ with $\Omega_{dipole} = \sec(\alpha)$ and $|r| = s$. Now, if we consider another regime with a broad function to represent the environment, so that it vanishes effectively for $x_2 = \lambda \gg d$, where $\lambda = \hbar / (M \kappa T)^{1/2}$,

$$\tau = \frac{\hbar^4 (M \kappa T)^{-3/2}}{3Kqps} \Omega_{dipole}. \quad (22)$$

Formula (22) shows that in the very low temperature regime our calculations for the dipole case yield a result compatible with the high decoherence time in the limit of very low temperature, as does Tegmark's [9]. The result we obtained is summarized in Table I, where we compare all the values obtained from our estimate as well as other estimates. Both are lower than the Tegmark [9] case and higher than Hagan, Hameroff, and Tuszynski [1] for dipole approximation.

VI. CONCLUSIONS

The result stated above points out that there is a big difference between the time period of coherence in the Hameroff-Penrose model [2,3] and the time of decoherence in the brain. We point out that their model must be modified when including decoherence in place of gravitational collapse. However, based on this difference, we do not conclude, as Tegmark [9] does, that the quantum approach to the brain problem is refuted if we use decoherence instead of gravitational collapse. The first point is that we must also consider the time for building coherence, while the system either remains relatively isolated to sustain coherence or there is no coherent collective state. Hameroff and Penrose speak about isolating microtubules from environmental entanglement but they did not take this point further.

We believe there is some kind of Fröhlich process acting in the brain sustaining the coherence [20]. In the Fröhlich theory the energy supplied to a nonlinear vibration system balanced by energy losses due to interaction with its surroundings is not immediately thermalized and can disturb thermodynamic equilibrium through energy condensation [10]. Depending on the energy supplied and on the energy transferred to the environment, the system shifts the regime from incoherent to time-limited coherent and in further steps to steady coherent.

Our result does not discard the conjecture that quantum theory can help us to understand the functioning of the brain, and maybe also to understand consciousness. We assume that the neuronal network in the brain processes information coming from the external environment or stored in the memory, working at a subneuronal level as a quantum system. Quantum coherent superposition of states is sustainable at the macroscopic level for a very short time, like a flash. When the physical system reaches a critical situation coherence is destroyed due to the decoherence process by interaction with the environment.

In spite of our disagreement with Tegmark [9] concerning his refutation of the quantum brain together with the Hameroff and Penrose OOR model [2,3], we have shown that our calculation does not agree with the response to Tegmark's paper by Hagan, Hameroff, and Tuszynski [1]. We still propose a new quantum model in the brain where the most important thing is the sequence of coherent states accumulating in the microtubule. In this manner, the quantum activity could appear in another formulation for the brain. The details of these considerations will be present in our future papers.

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